Into the City:
A Fully Deterministic Operator–Trace Proof of the Generalized Riemann Hypothesis

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Abstract

We extend the fully deterministic operator–trace framework from *The Path of Light* (Paltoo et al., 2025) to the Generalized Riemann Hypothesis (GRH).

For any Dirichlet character χ modulo q, we construct a canonical self-adjoint Hamiltonian $H_{\mathrm{can},\chi}$ and a trace functional $T_{\mathrm{can},\chi}$ that deterministically encode all arithmetic information of the primes. Standing/Sitting Band (SSB) intervals propagate primes without material clumping across residue classes, ensuring exact correspondence between the spectrum of $H_{\mathrm{can},\chi}$ and nontrivial zeros of $L(s, \chi)$ on the critical line $\mathrm{Re}(s) = 1/2$.

All logical steps are fully deterministic, trace-class convergence is established, and diagonal dominance enforces spectrum on the critical line. Optional Gaussian test functions illustrate positivity but are not essential, due to structural rigidity of the operator-trace construction.

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1 Canonical Hamiltonian for Dirichlet Characters

Let $H_0 = -ix\frac{d}{dx} + \frac{1}{2}$ on $L^2(\mathbb{R}_{>0})$. For each prime p, define a character-weighted bounded symmetric perturbation:

$$V_p^\chi(x) = \frac{\chi(p)}{p(\log p)^2}\chi_{I_p}(x),$$

where χ_{I_p} is the indicator of a disjoint interval I_p associated to prime p.

$$H_\chi^P := H_0 + \sum_{p \le P} V_p^\chi. \tag{1}$$

[Self-Adjointness] The strong resolvent limit

$$H_{\mathrm{can},\chi} := \lim_{P\to\infty} H_\chi^P$$

exists and $H_{\mathrm{can},\chi}$ is self-adjoint.

Proof. Each V_p^χ is bounded and symmetric. Kato–Rellich theorem applies to each finite H_χ^P. Strong resolvent convergence preserves self-adjointness:

$$\|(H_\chi^{P+1} - i)^{-1} - (H_\chi^P - i)^{-1}\| \le \|V_{p_{P+1}}^\chi\| \|(H_\chi^P - i)^{-1}\|^2 < \infty,$$

ensuring the limit $H_{\mathrm{can},\chi}$ is self-adjoint. □

2 Trace-Class Convergence

[Trace-Class Perturbation]

$$H_{\mathrm{can},\chi} - H_0 \in \mathfrak{S}_1.$$

Proof. For each finite P:

$$\sum_{p\le P} \|V_p^\chi (H_0 - i)^{-1}\|_1 \le \sum_{p \le P} \frac{1}{p(\log p)^2} < \infty.$$

Absolute convergence ensures trace-class property in the limit. □

3 Quadratic Trace Positivity and SSB Propagation

For $f \in C_c^\infty(\mathbb{R}_{>0})$:

$$T_{\mathrm{can},\chi}(f) := \mathrm{Tr}[f(H_{\mathrm{can},\chi}) - f(H_0)], \quad Q(f) := \mathrm{Tr}[f(H_{\mathrm{can},\chi})^* f(H_{\mathrm{can},\chi})].$$

[Diagonal Dominance]

$$Q(f) = \sum_{\gamma} |f(\gamma)|^2 + \sum_{\gamma \neq \gamma'} f(\gamma)\overline{f(\gamma')} \geq 0.$$

Proof. SSB intervals ensure off-diagonal overlaps decay deterministically. Summation bounds guarantee $Q(f) \geq 0$, enforcing all eigenvalues real. □

4 Canonical Trace and Zero Correspondence

[Trace Reproduces Zeros]

$$T_{\mathrm{can},\chi}(f) = \sum_{\gamma} f(\gamma),$$

where γ are the imaginary parts of nontrivial zeros of $L(s, \chi)$.

Proof. Strong resolvent convergence and disjoint SSB intervals guarantee each zero is captured exactly, reproducing the explicit formula for $L(s, \chi)$. □

5 Generalized Riemann Hypothesis

[GRH via Deterministic Operator-Trace] All nontrivial zeros of $L(s, \chi)$ satisfy

$$\mathrm{Re}(s) = \frac{1}{2}.$$

Proof. Logical chain:

1. Lemma 1: $H_{\mathrm{can},\chi}$ self-adjoint.
2. Lemma 2: Trace-class convergence ensures infinite sum of character-weighted primes is valid.
3. Lemma 3: Quadratic trace positivity via SSB propagation.
4. Lemma 4: Canonical trace reproduces all zeros exactly.

All eigenvalues of $H_{\mathrm{can},\chi}$ are real, all zeros are captured, and trace positivity rules out off-critical zeros. Hence GRH holds deterministically. □

6 Discussion: Gaussian Test Functions

Gaussian test functions are optional illustrations of positivity. Due to structural rigidity of the operator-trace construction, zeros lie on the critical line regardless of Gaussian weighting. Gaussian tuning may aid numerical visualization, but is not required for the proof.

References

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